Algebraic Linear Orderings

An algebraic linear ordering is a component of the initial solution of a first-order recursion scheme over the continuous categorical algebra of countable linear orderings equipped with the sum operation and the constant 1. Due to a general Mezei-Wright type result, algebraic linear orderings are exactly those orderings isomorphic to the linear ordering of the leaves of an algebraic tree. Using Courcelle's characterization of algebraic trees by their branch languages we obtain a description of algebraic linear orderings by deterministic context-free languages. When the algebraic linear ordering is a well-ordering, its order type is an algebraic ordinal.

We prove that the Hausdorff rank of any scattered algebraic linear ordering is less than ω^ω. It follows that the algebraic ordinals are exactly those less than ω^{ω^ω}. We also cover some decision problems.